C. ⑫

Physica Scripta. Vol. 39, 128–132, 1989.

Spatio-Temporal conditions for Apparent Movement

C. Bundesen

Psychological Laboratory, Copenhagen University, Njalsgade 90, DK-2300 Copenhagen S, Denmark

Received February 2, 1987; accepted February 1, 1988

Abstract

A number of psychophysical studies on apparent translatory movement, apparent rotational movement, and compositions of these transformations are reviewed. The studies have yielded modern equivalents and extensions of Korte's third law. The results support the hypotheses that (a) apparent movements over short and long distances are processed by separate mechanisms and (b) apparent movement over a long distance is represented by impleting ("filling in") a path connecting the discrete stimulus presentations. It is suggested that impletion serves economy of processing by normalizing the internal representation of an object viewed in apparent movement to the format used for objects viewed in real movement.

1. Introduction

When a stationary object is presented first in one position and, after an appropriate time interval, again in a second position, the human observer gets a visual impression of a single object moving smoothly across the field from the first position to the second one. The phenomenon is called "apparent movement" to emphasize that there is no physical movement, but merely an impression of movement.

Apparent movement was first subjected to intense and detailed study by Max Wertheimer, the founder of Gestalt psychology. Wertheimer's pioneering work [1] instigated a number of psychophysical studies exploring the spatio-temporal conditions for the phenomenon. In this article, I summarize a few of these studies. Following the presentation of the data, I sketch a theoretical interpretation in terms of internal transformations of visual representations. The interpretation, too, may be traced back to Wertheimer. The current version attempts to link up ideas stemming from the classical approach to apparent movement with ideas coming from the more recent "computational approach" exemplified by the work of Ullman [2].

2. Parametric studies

2.1. *Apparent translation*

Perhaps the simplest case of apparent movement is seen when two laterally separated spots of light are presented in succession. When the timing is appropriate, the observer gets a visual impression of a single spot moving smoothly ("continuously") over the straight path between the two positions of presentation. The most important temporal parameters are the interstimulus interval and the stimulus-onset asynchrony. The *interstimulus interval* (ISI) is the time interval from the offset of the first stimulus to the onset of the second one. If the ISI is increased up to the "succession threshold", the impression of a single spot in smooth motion gives way to an impression of two different, successive spots, with or without some motion. The *stimulus-onset asynchrony* (SOA) is the time interval from the onset of the first stimulus to the onset of the second one. If the SOA is decreased down to the "simultaneity threshold", the impression of a single spot in smooth motion gives way to an impression of two different, simultaneous or nearly simultaneous spots.

In 1915, Korte [3] explored the temporal conditions for the illusion of apparent continuous translation of a spot of light over spatial distances ranging from about 2 deg to nearly 15 deg of visual angle. He hypothesized from his results that, when stimulus intensity and duration are held constant, the ISI required for apparent continuous movement ("optimal apparent movement" or "beta movement") is directly related to the spatial separation between the stimuli. The hypothesis became known as the third law of Korte, but it did not stand the test of time. Parametric studies by Neuhaus [4] and others [5–8] showed that if a given ISI provides beta movement over a given spatial separation, it provides beta movement over any smaller separation too. Nevertheless, Korte's third law does appear to embody an important principle. The cited studies suggest that when stimulus intensity and duration are kept constant, the minimum SOA required for apparent movement (the simultaneity threshold for apparent continuous translation) increases with the spatial separation.

Corbin [9] attempted to dissociate effects of retinal separation from effects of apparent separation in three-dimensional space by changing the slant of the screen on which his stimuli were positioned; he found that the simultaneity threshold for apparent movement depended upon separation in three-dimensional space with no effect of retinal separation per se. In a related study by Attneave and Block [10], goodness of apparent movement was determined jointly by retinal separation and physical separation in three-dimensional space.

Corbin himself did not analyze the functional form of the relationship between spatial separation and threshold for apparent movement, but his data for apparent movement over visual angles of about one deg or more suggested a remarkably simple relationship: The minimum SOA required for apparent continuous translation was approximately a linearly increasing function of the apparent distance between the stimuli in three-dimensional space. To test the generality of this relationship, Larsen, Farrell, and Bundesen [11] presented subjects with two point sources in sequential alternation (with zero ISI and zero intercycle interval) and measured the minimum SOA required for apparent continuous translation as a function of the (lateral) separation of the sources with separations ranging from about one tenth of a degree up to 5 deg of visual angle. Viewing distance was varied as a parameter. With viewing distance kept constant, the minimum SOA required for apparent continuous translation increased with the angular separation of the sources over the entire range of visual angles investigated. As illustrated in Fig. 1, the function showed a strong linear increase over angles less than about a quarter of a degree (short-range

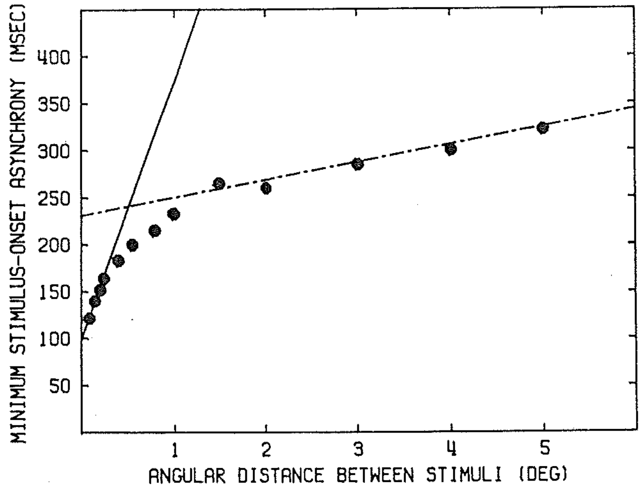

Fig. 1. Minimum stimulus-onset asynchrony required for apparent translation (beta movement) as a function of the visual angle separating the stimuli. Separate straight lines are fitted to the data for small (⩽ 0.25 deg) and large (⩾ 1.5 deg) visual angles.

component of the function) and a much weaker linear increase over angles greater than about one deg (long-range component). The short- and long-range components of the function were differentially affected by a fourfold increase in viewing distance (see Fig. 2): The long-range component increased in slope, but the extrapolated zero-intercept was constant; the short-range component changed in intercept, but not in slope.

Our results supported a two-process theory for apparent movement proposed by Braddick [12, 13] and Anstis [14, 15] on the basis of other types of psychophysical and neurophysiological findings. In this theory, visual impressions of movement may result from either of two processes, a central process that underlies apparent movement over large visual angles ("long-range apparent movement") and a more peripheral process that produces impressions of movement over small visual angles ("short-range apparent movement" or "real movement"). Long-range movement is thus supposed to be indirectly inferred by a high-level interpretive process; short-range movement is supposed to be sensed directly by lower-level motion-detecting mechanisms. In fact, for apparent movement over large visual angles, our data showed that the rate of increase in the simultaneity threshold with the angular separation of the stimuli increased with viewing distance and, accordingly, with the apparent separation of the stimuli in three-dimensional space. This result compares with the data of Corbin [9] and Attneave and Block [10], suggesting that long-range movement is signalled by a process that operates on the output from prior computations of the positions of the stimuli in three-dimensional space [16]. For apparent movement over small visual angles, the rate of increase in the simultaneity threshold with the angular separation of the stimuli was independent of viewing distance. This result would be expected from the hypothesis that the short-range process, being more peripheral, responds to the retinal separation of stimuli without regard to their apparent depth.

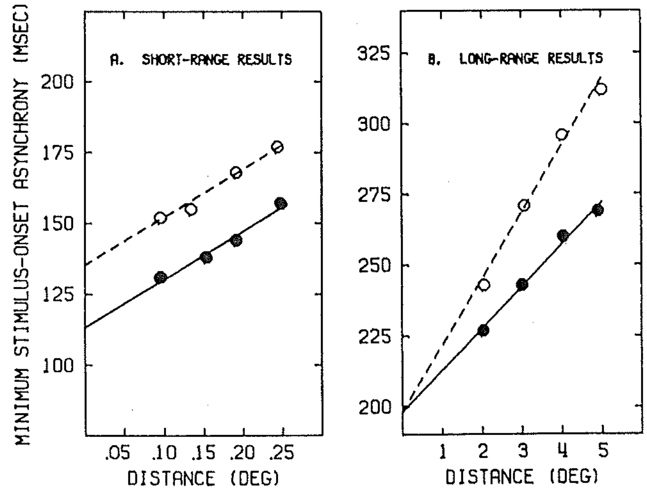

Fig. 2. Minimum stimulus-onset asynchrony required for apparent translation (beta movement) as a function of the visual angle separating the stimuli with viewing distance as the parameter. Viewing distance was 1.5 m (closed circles and solid lines) or 6 m (open circles and dashed lines). (A) Results for small visual angles fitted by a pair of parallel lines. (B) Results for large visual angles fitted by a pair of straight lines with a common zero-intercept.

2.2. *Apparent rotation*

Wertheimer [1] presented his subjects with a view of a line segment in one orientation followed by a view of the same line segment in another orientation. When timing was appropriate, the presentation produced a visual impression of a single, rigid line segment rotating smoothly through the smallest angle between the two orientations of presentation. Many years later, Shepard and Judd [17] investigated the spatio-temporal conditions for this illusion by presenting a pair of perspective views of a three-dimensional object in sequential alternation and measuring the minimum SOA required to generate a visual impression of the three-dimensional object in rigid rotation back and forth between the two orientations. The minimum SOA was found to be a linearly increasing function of the angular extent of the transformation, and the function was nearly the same for rotations in depth as for rotations in the picture plane.

Since Shepard and Judd did not vary the size of their stimulus objects, the angular extent of the apparent transformations of the stimulus objects was perfectly correlated with the linear extent of the paths apparently traversed by those points or elements of the stimuli that were farthest away from the center of rotation. A later study by Farrell, Larsen, and Bundesen [18] separated effects of angular displacement from effects of linear displacement by showing that the slope of the function relating the SOA threshold for apparent rigid rotation to the angular extent of the transformation was virtually the same regardless of the size of the stimulus object (see Fig. 3).

2.3. *Translation and rotation combined*

Bundesen, Larsen, and Farrell [19] presented subjects with two sequentially alternating stimulus patterns that were the same in shape, but differed in size and angular orientation in the frontoparallel picture plane. When the rate of alternation was sufficiently slow, the presentation produced a visual impression of a rigid object moving in three-dimensional space. If stimulus size was kept constant, the object appeared to be rotating back and forth in a frontoparallel plane. When stimulus size was changing, the object appeared to be moving to and fro in depth rather than changing in distal size. Apparent changes in distance and orientation seemed to be gradual and concurrent in the sense that, when the object

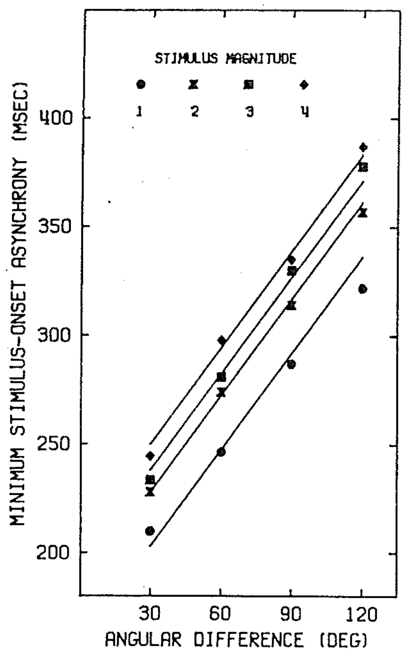

Fig. 3. Minimum stimulus-onset asynchrony required for apparent rotation as a function of the angular difference in orientation between the stimuli with stimulus size as the parameter. Data are fitted by four parallel lines.

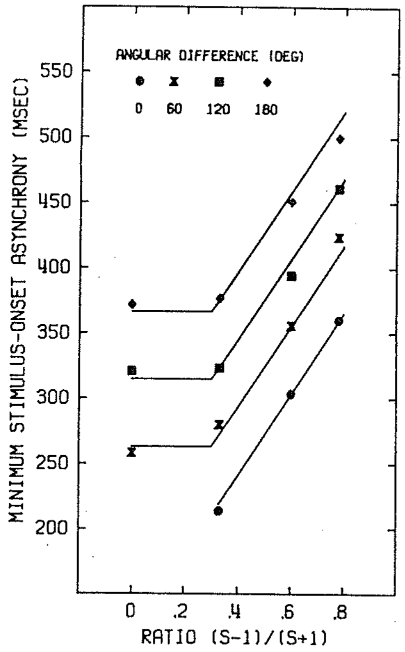

Fig. 5. Minimum stimulus-onset asynchrony required for apparent translation in depth combined with rotation as a function of $(s - 1)/(s + 1)$ and v, where s is the linear size ratio between stimuli and v is the angular difference in orientation. A theoretical fit is indicated by solid curves.

appeared at intermediate distances, its apparent angular orientation was intermediate between those of the two stimuli.

The subjective reports suggested that changes in stimulus size were visually resolved as changes in apparent distance such that, within trials, the apparent distal size of the moving object was approximately constant. To analyze the situation geometrically, let x and y be alternating stimulus patterns presented in a frontoparallel plane at a distance D from the subject (see Fig. 4). Let L_x and L_y, where $L_x \leqslant L_y$, be the linear sizes (say, the greatest linear extents) of x and y, and let s be the size ratio L_y/L_x. If it is assumed that the visual system acts as "a projective decoder of the optical message" [20], complete visual resolution of the size disparity between x and y as a difference in depth should correspond to central projection of the patterns onto frontoparallel planes at distances d_x and d_y, respectively, such that apparent distal sizes l_x and l_y are equalized. It is easy to show geometrically that $l_x = l_y$ if, and only if,

$$d_x = sd_y. \tag{1}$$

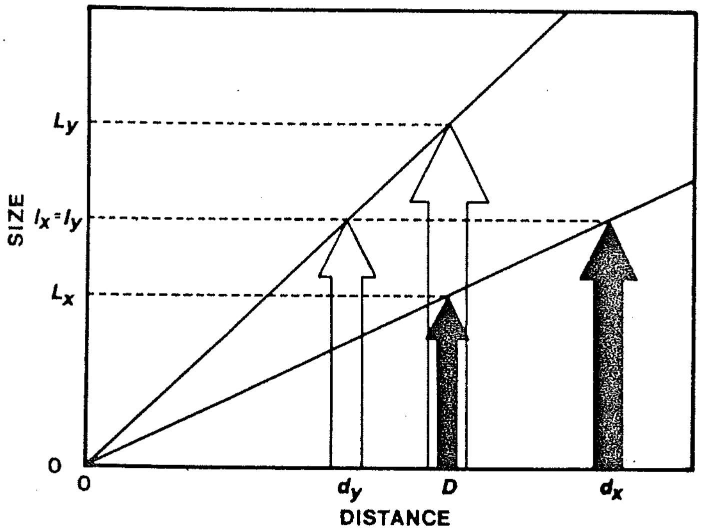

Fig. 4. Resolving a disparity of size as a difference in depth. Objects x (smaller solid arrow) and y (larger open arrow) with linear sizes L_x and L_y are shown at a distance D from a subject. By geometric multiplications with respect to the subject's point of view (the origin of the coordinate system), the disparity of size between x and y is resolved as a difference in depth between the images of x (larger solid arrow) and y (smaller open arrow). The sizes and distances of the images are designated by l_x, l_y and d_x, d_y.

There are infinitely many pairs of distances d_x and d_y that satisfy eq. (1). However, given the resulting visual impression of a single object in movement, a plausible constraint on the visual resolution of the disparity of size as a difference in depth is that the average apparent distance of that object, $(d_x + d_y)/2$, be some constant a, regardless of L_x and L_y. The constant average apparent distance a should be determined by the constant indications of distance from accommodation, convergence, and binocular disparity; the deviation from a represented by $d_x - d_y$ should be a function of the sole cue to such a difference in depth, the disparity of size between x and y.

By use of eq. (1), it is easy to verify that

$$d_x - d_y = (d_x + d_y)(s - 1)/(s + 1). \tag{2}$$

Equation (2) implies that, if the average apparent distance $(d_x + d_y)/2$ is a constant, then the difference in apparent depth between the stimulus patterns x and y is directly proportional to the ratio $(s - 1)/(s + 1)$.

The results of Bundesen *et al.* [19] are illustrated in Fig. 5 which shows the SOA threshold for the appearance of a rigidly moving object as a function of the ratio $(s - 1)/(s + 1)$ with the angular extent of the transformation v as the parameter. The SOA threshold was approximately a linearly increasing function of v for simple rotation,

$$\mathrm{SOA}_{\mathrm{rot}} = \alpha + \beta v, \tag{3}$$

and a linearly increasing function of the ratio $(s - 1)/(s + 1)$ for simple translation in depth,

$$\mathrm{SOA}_{\mathrm{trans}} = \gamma + \delta\,(s - 1)/(s + 1), \tag{4}$$

and the extrapolated zero-intercept α of the threshold function for apparent rotation was higher than the extrapolated zero-intercept γ of the threshold function for apparent translation in depth. For composite transformations, the SOA threshold was fitted by

$$\mathrm{SOA_{comp}} = \begin{cases} \mathrm{SOA_{trans}} + \beta v, & \text{if } \mathrm{SOA_{trans}} \geqslant \alpha \\ \alpha + \beta v, & \text{if } \mathrm{SOA_{trans}} < \alpha. \end{cases} \tag{5}$$

As indicated in Fig. 5, the fit was fairly good.

3. General discussion

We have seen that, for apparent movement over long distances, the minimum SOA required for apparent continuous translation depends upon the apparent separation between the stimuli in three-dimensional space. This finding suggested that long-range apparent movement is signalled by a central process that operates on the output from prior computations of the positions of the stimuli in three-dimensional space. More specifically, the procedure for generating long-range apparent movement is assumed to include three stages of processing:

First, given two successive images of an object that are widely separated in space and time, the visual system identifies corresponding parts of the two images. Second, when the "correspondence problem" has been solved, the system computes the three-dimensional structure of the object and the motion intervening between the two presentations [2]. Finally, the computed trajectory is *impleted* [21] or "filled in" by generation of a sequence of visual representations of the object in successive positions along the path from the position indicated by the first image to the position indicated by the second one – a sequence of representations similar to those that would have been generated if the object had been viewed in real movement over the trajectory.

The long-range process preferentially generates impressions of *rigid* movement [2, 20, 22]. Rigid movements are decomposable into translations and rotations, and a translation is decomposable into smaller translations whereas a rotation is decomposable into smaller rotations. Thus, a simple, general procedure for generating a sequence of visual representations of an object in rigid movement might consist in iterating one routine that transforms a visual representation of an object into a visual representation of the same object after an incremental translation (a translation over a certain small distance) and another routine that transforms a visual representation of an object into a visual representation of the same object after an incremental rotation (a rotation over a certain small angle).

Suppose that long-range apparent movement of rigid objects is impleted in successive small steps such that each step is an incremental translation or an incremental rotation. The main features of our parametric data on SOA thresholds may then be explained by assuming that the minimum time required to perform an incremental transformation (translation or rotation) is a constant regardless of the sequence of transformations in which it is embedded:

In the suggested interpretation, the minimum impletion time for apparent translation is directly proportional to the linear extent of the transformation; the minimum impletion time for apparent rotation is directly proportional to the angular extent of the transformation; and the impletion time for a composite transformation of apparent translation and rotation equals the sum of the impletion times for the components. Thus, in eqs. (3) and (4), slope constants β and δ are measures for the maximum speed of impleting apparent rotation and apparent translation, respectively. The zero-intercepts α and γ of the threshold functions measure the time taken to establish correspondence and specify the trajectory to be impleted. Intercept α for simple rotation was higher than intercept γ for simple translation, and eq. (5) means that, in composite transformations too, impletion of apparent translation was released with the shorter latency γ whereas impletion of apparent rotation was released with the longer latency α.

The impletion interpretation proposed for long-range apparent movement explains the main features of our parametric data on SOA thresholds. The explanation raises an obvious question. Given that impletion of apparent movement does not occur until the trajectory to be impleted has already been computed, why does impletion occur at all?

The purpose of impletion, I think, is to decrease the demands on later stages of processing by normalizing the internal representation of objects viewed in apparent movement (i.e., with discrete presentations) to the format used for objects viewed in real movement. Following normalization of long-range discrete input to real-movement format, the scene analysis and recognition routines used in analyzing representations of objects viewed in real movement may be directly applied to representations of objects seen in apparent movement. Thus, considering the visual system as a whole, the impletion stage in long-range apparent movement may serve economy of processing.

References

1. Wertheimer, M., Z. Psychol. **61**, 161 (1912).
2. Ullman, S., The Interpretation of Visual Motion, MIT Press, Cambridge, Mass. (1979).
3. Korte, A., Z. Psychol. **72**, 193 (1915).
4. Neuhaus, W., Arch. G. Psychol. **75**, 315 (1930).
5. Caelli, T., Hoffman, W. C. and Lindman, H., Formal Theories of Visual Perception (Edited by E. L. J. Leeuwenberg and H. F. J. M. Buffart), p. 103, Wiley, New York (1978).
6. Caelli, T. and Finlay, D., Perception **8**, 59 (1979).
7. Finlay, D. and Caelli, T., Perception **8**, 595 (1979).
8. Caelli, T. and Finlay, D., Perception **10**, 183 (1981).
9. Corbin, H. H., Arch. Psychol. **273**, 1 (1942).
10. Attneave, F. and Block, G., Percept. Psychophys. **13**, 301 (1973).
11. Larsen, A., Farrell, J. E. and Bundesen, C., Psychol. Res. **45**, 11 (1983).
12. Braddick, O., Vision Res. **14**, 519 (1974).
13. Braddick, O., Phil. Trans. R. Soc. Lond. B **290**, 137 (1980).
14. Anstis, S. M., Handbook of Sensory Physiology (Edited by R. Held, H. W. Leibowitz and H. -L. Teuber), Vol. VIII, p. 655, Springer, Berlin (1978).
15. Anstis, S. M., Phil. Trans. R. Soc. Lond. B **290**, 153 (1980).
16. Our data resemble those of Attneave and Block [10] by showing effects of both retinal and physical three-dimensional separation. If the simultaneity threshold for apparent translation were determined solely by the apparent three-dimensional separation between stimuli, and the apparent distances equalled the physical ones, the ratio of the slope constants for the long-range functions in Fig. 2 (1 : 1.6) should approximate the ratio of the two viewing distances (1 : 4). The disparity between these ratios suggests that either the apparent distances in three-dimensional space differed from the physical ones or else the threshold for long-range movement was determined jointly by the angular and the apparent three-dimensional separation between stimuli.
17. Shepard, R. N. and Judd, S. A., Science **191**, 952 (1976).
18. Farrell, J. E., Larsen, A. and Bundesen, C., Perception **11**, 541 (1982).

19. Bundesen, C., Larsen, A. and Farrell, J. E., Perception **12**, 549 (1983).
20. Johansson, G., Formal Theories of Visual Perception (Edited by E. L. J. Leeuwenberg and H. F. J. M. Buffart), p. 265, Wiley, New York (1978).
21. Beck, J., Elsner, A. and Silverstein, C., Percept. Psychophys. **21**, 33 (1977).
22. Farrell, J. E. and Shepard, R. N., J. Exp. Psychol.: Hum. Percept. Perform. **7**, 477 (1981).